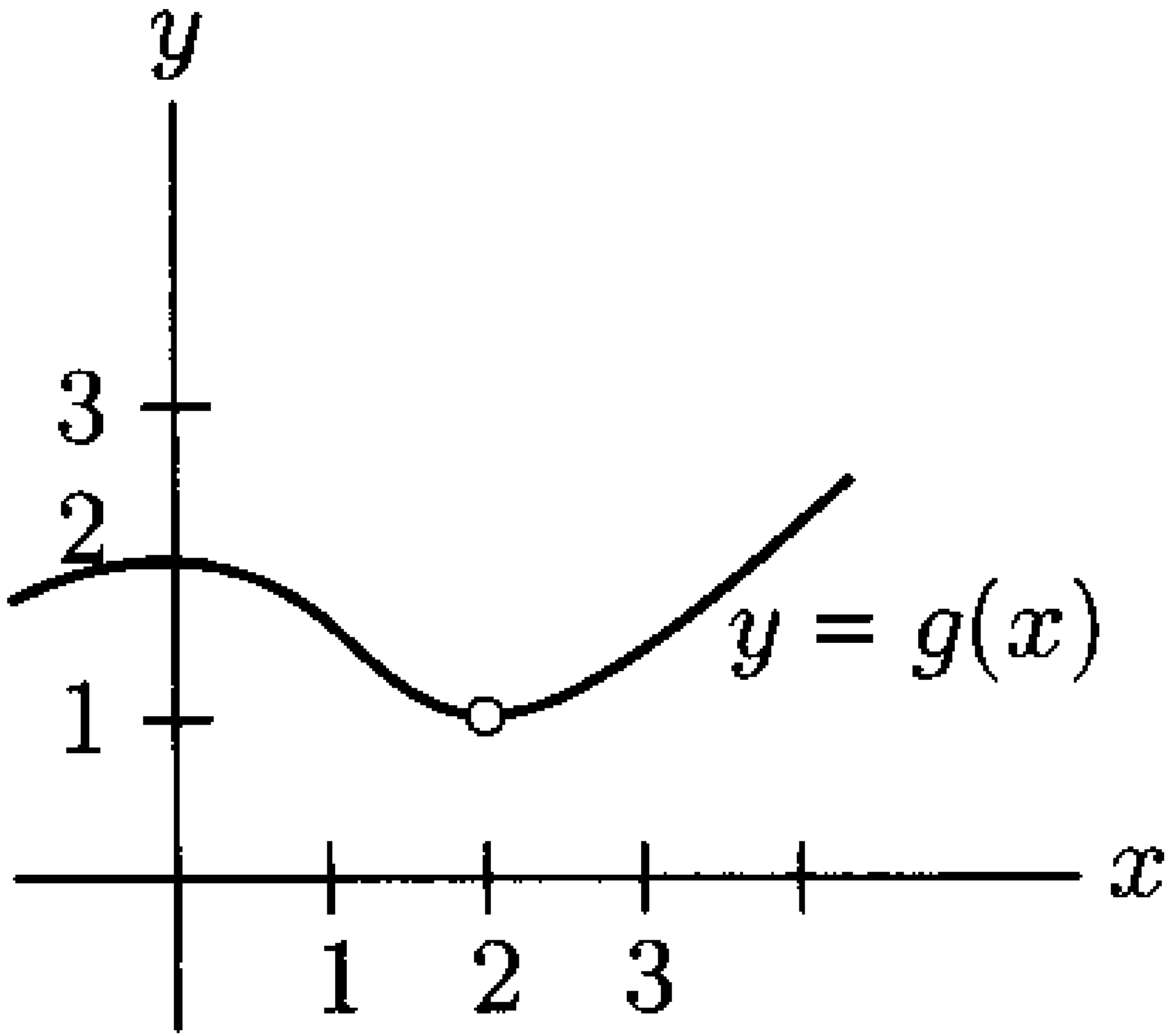
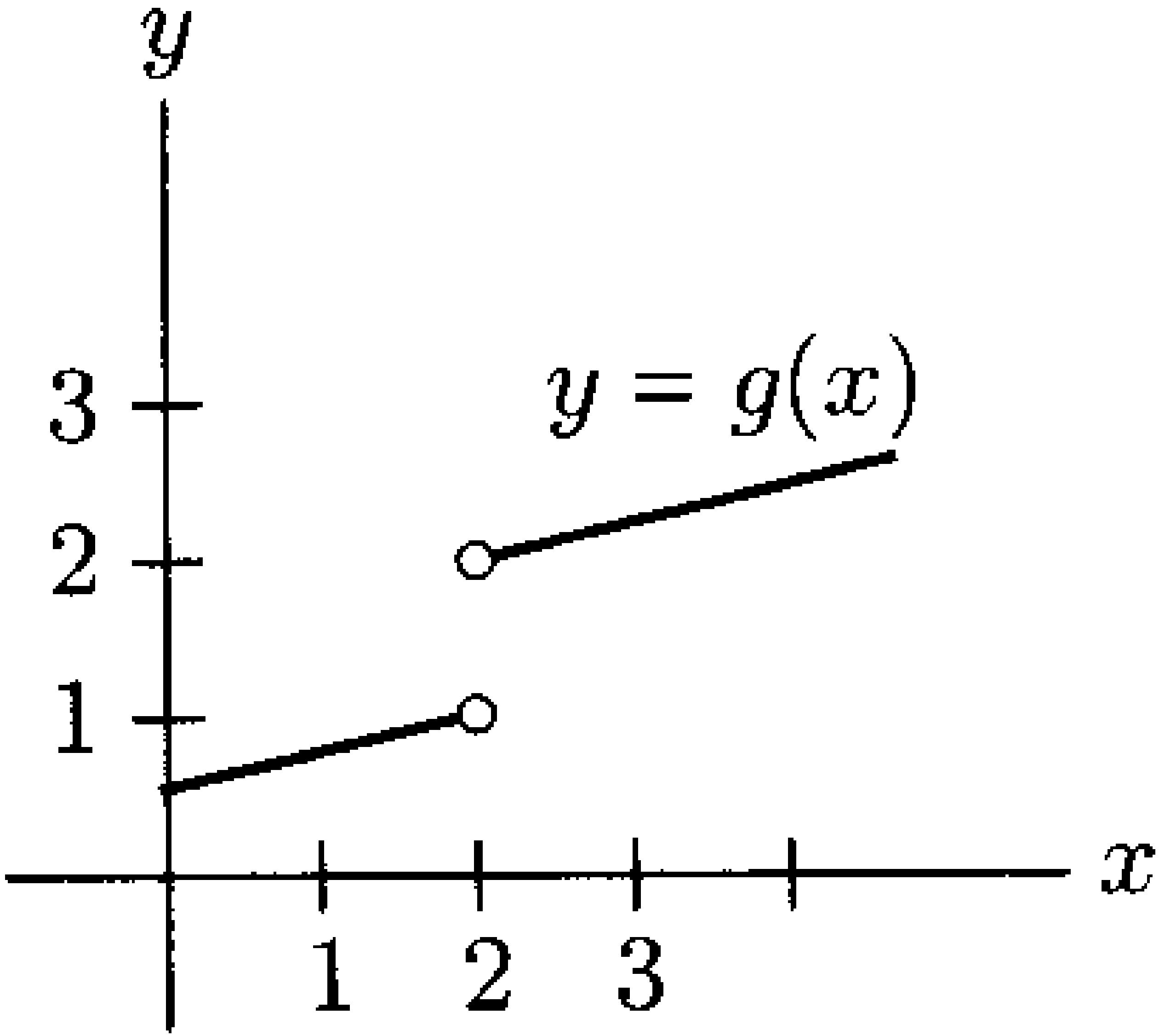
Determine if

$$\lim_{x \to 2} g(x)$$

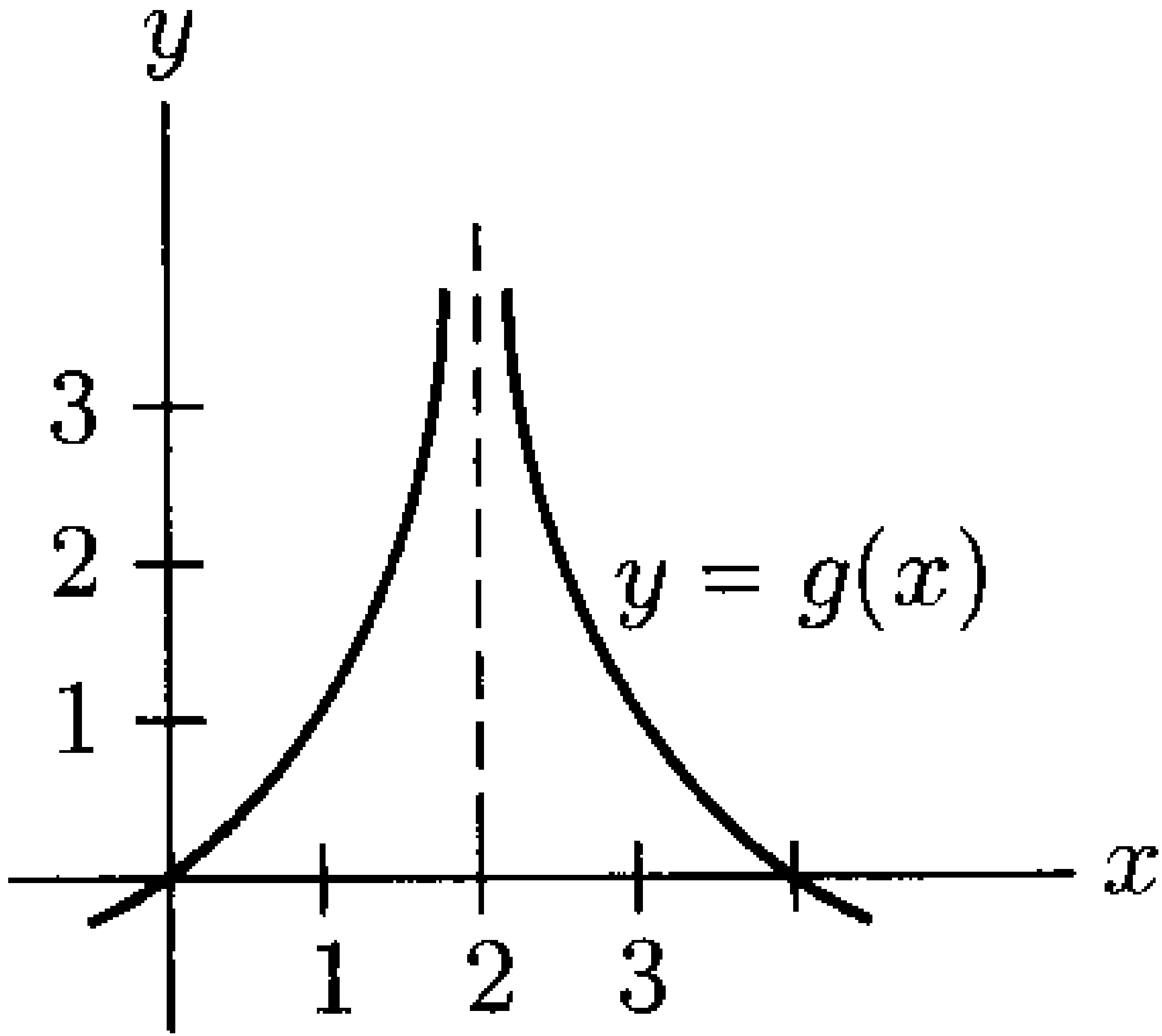
exists for the functions in the following graphs



(a)



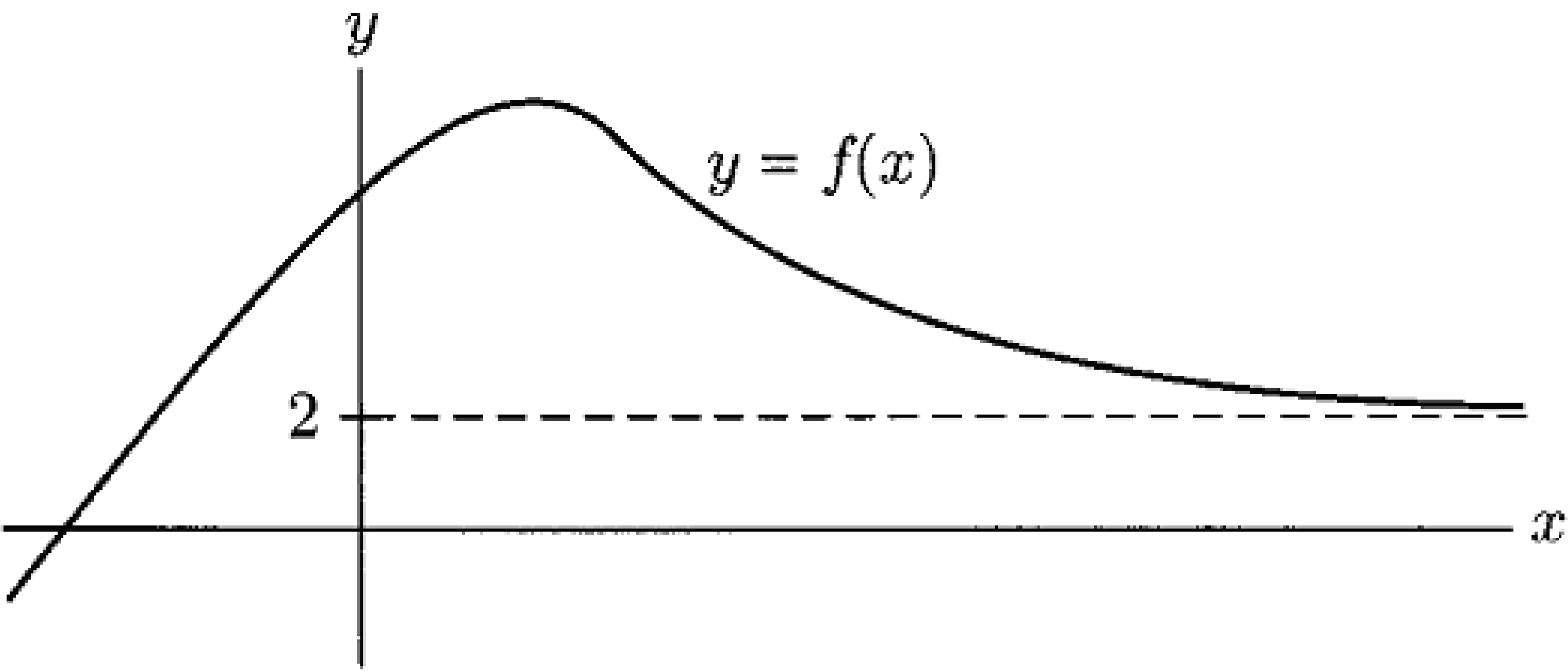
(b)



(c)

Infinity and Limits

Consider the function f(x) whose graph is



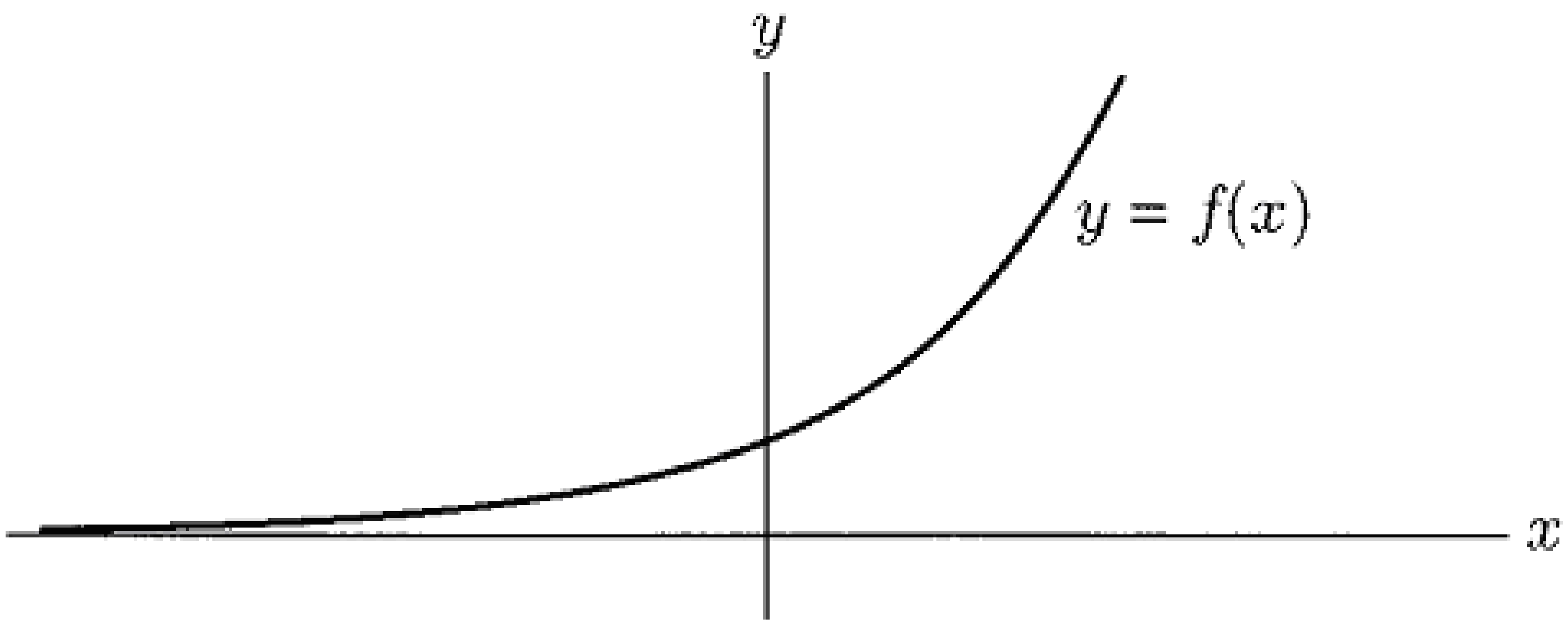
As x increases, the value of f(x) approaches 2.

In this case, we say that the limit of f(x) as x approaches infinity is 2.

We express this using limit notation as

$$\lim_{x \to \infty} f(x) = 2$$

Similarly, if we examine the following graph, we will note that as x grows large in the negative direction, the value of f(x) approaches 0.



We express this using limit notation as

$$\lim_{x \to -\infty} f(x) = 0$$

Example

$$\lim_{x \to \infty} \left(\frac{1}{x^2 + 1} \right)$$

1.5 Differentiability and Continuity

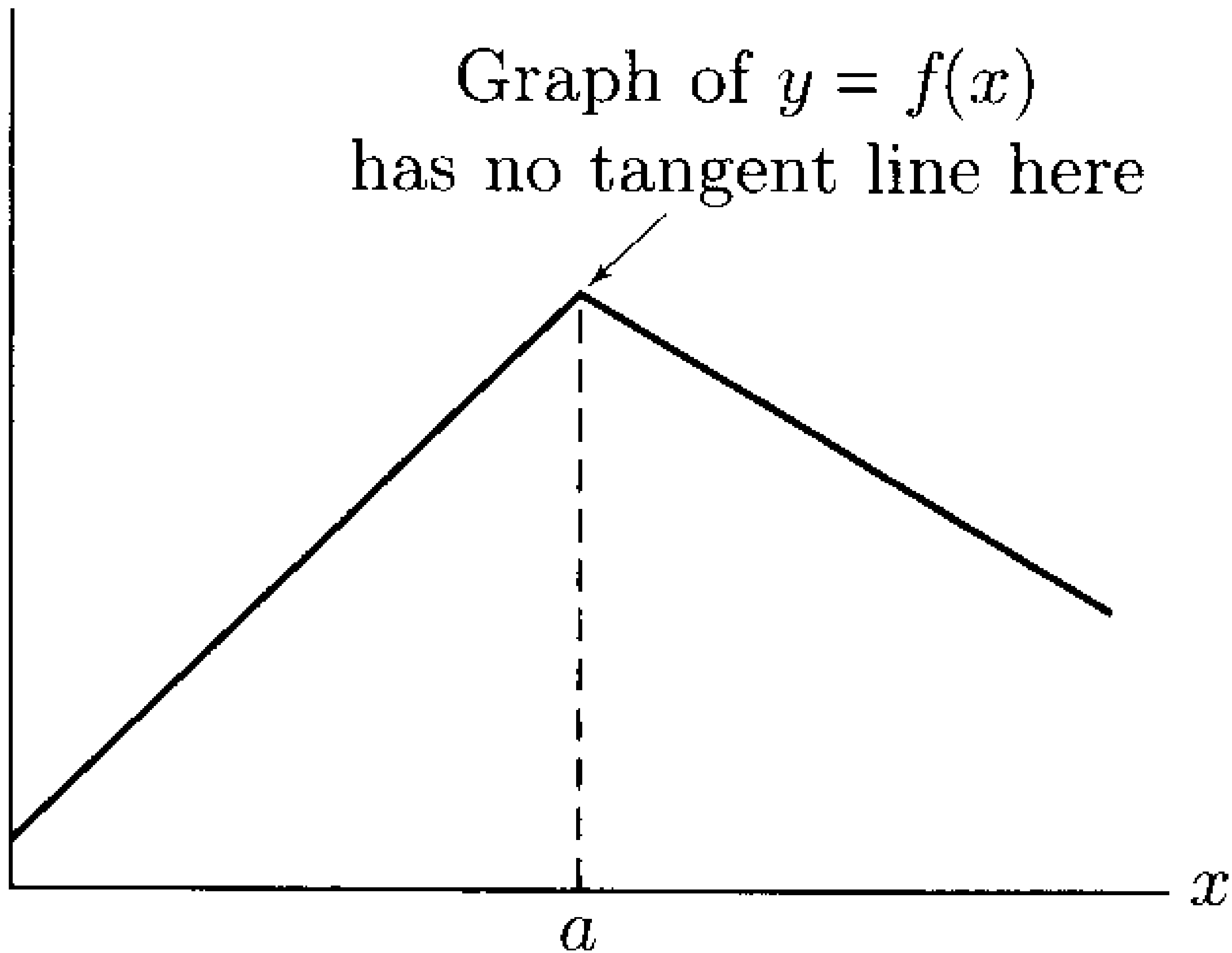
If a is a constant, we say that f(x) is differentiable at x = a if we can evaluate the following limit to determine f ′(a).

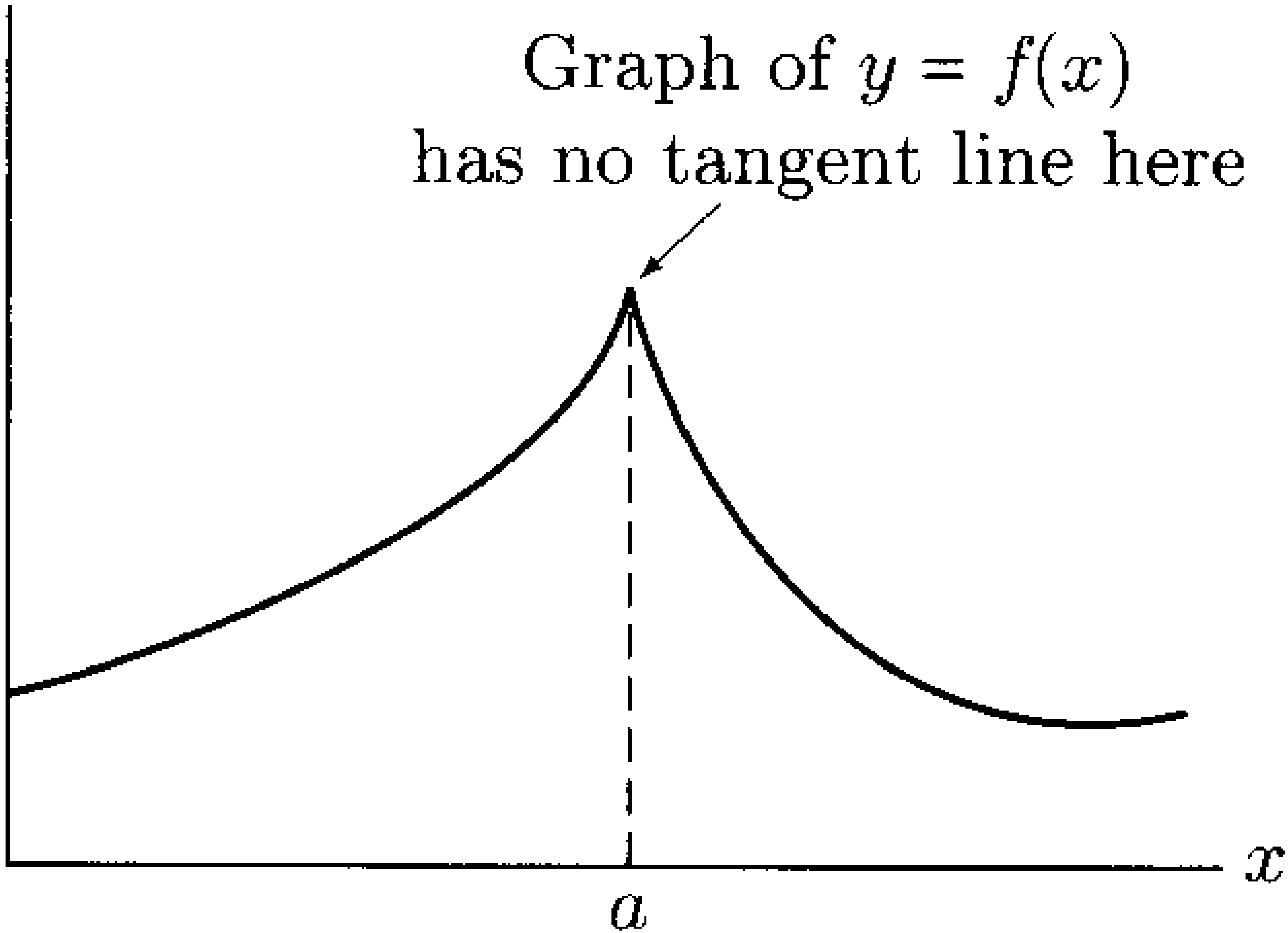
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

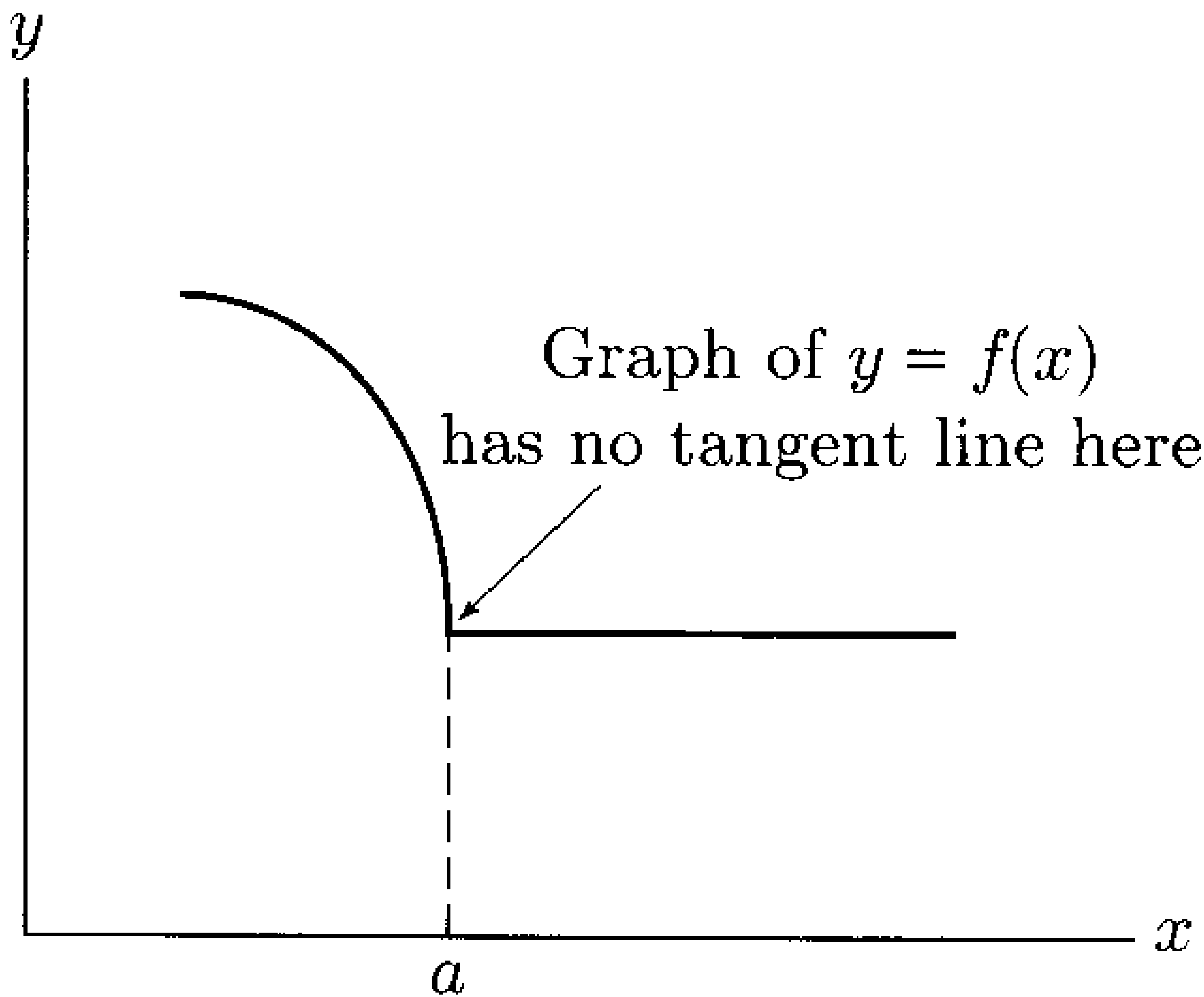
Conversely, if this limit does not exist, then f(x) is nondifferentiable at x = a.

There are many geometric representations of $f(x)$ for functions that are nondifferentiable at $x = a$.

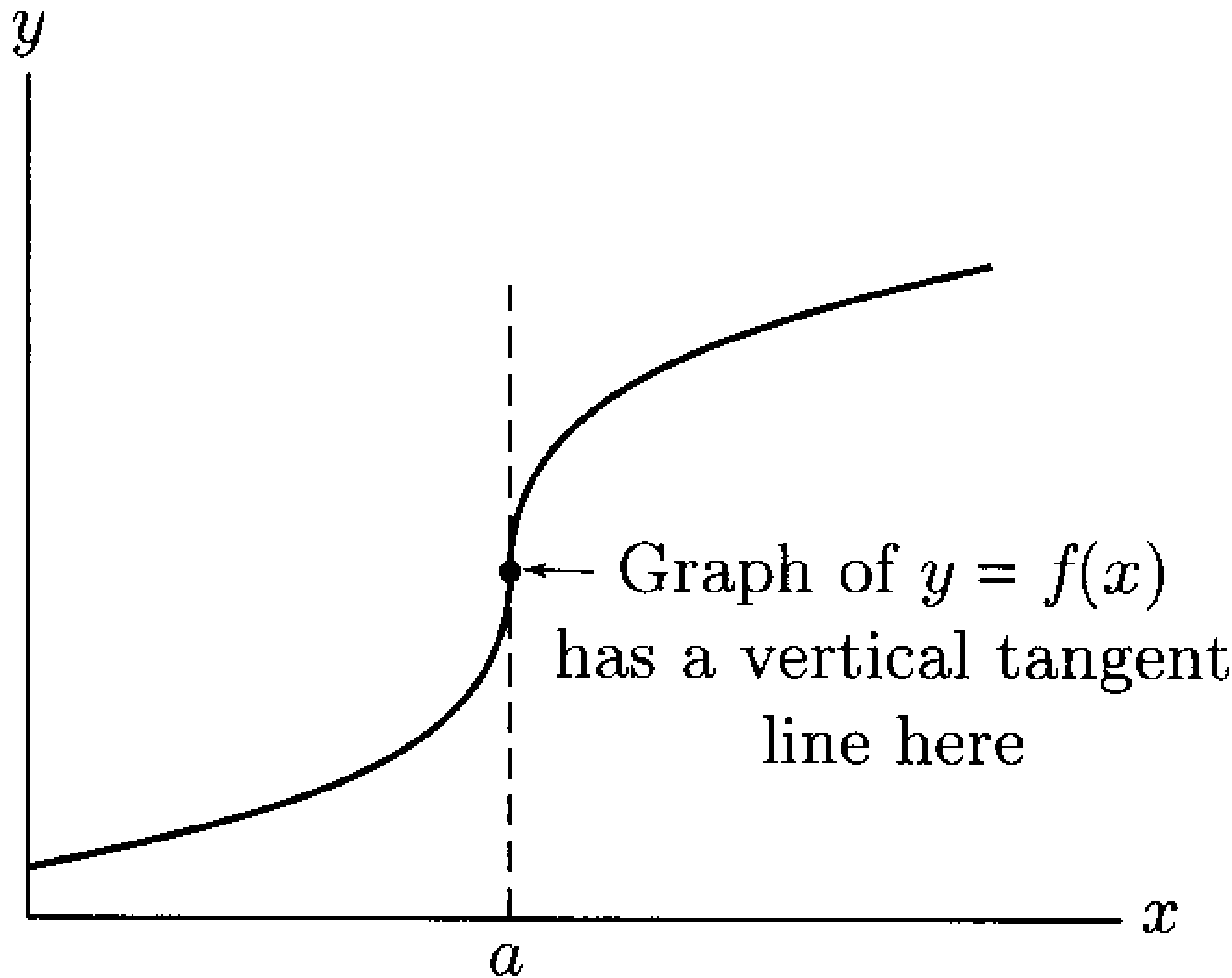
These can result if $f(x)$ has no tangent line at $x = a$, or if $f(x)$ has a vertical tangent line at $x = a$.


Graph of $y = f(x)$
has no tangent line here
x
a


Graph of $y = f(x)$
has no tangent line here
x
a



y
Graph of $y = f(x)$
has no tangent line here
x
a


y
x
a
Graph of $y = f(x)$
has a vertical tangent
line here

A railroad company charges \$10 per mile to haul a boxcar up to 200 miles and \$8 per mile for each mile exceeding 200. In addition, the railroad charges a \$1000 handling charge per boxcar.

Graph the cost of sending a boxcar x miles.

If x is at most 200 miles, then the cost C(x) is given by:

$$C(x) = 1000 + 10x \text{ dollars}$$

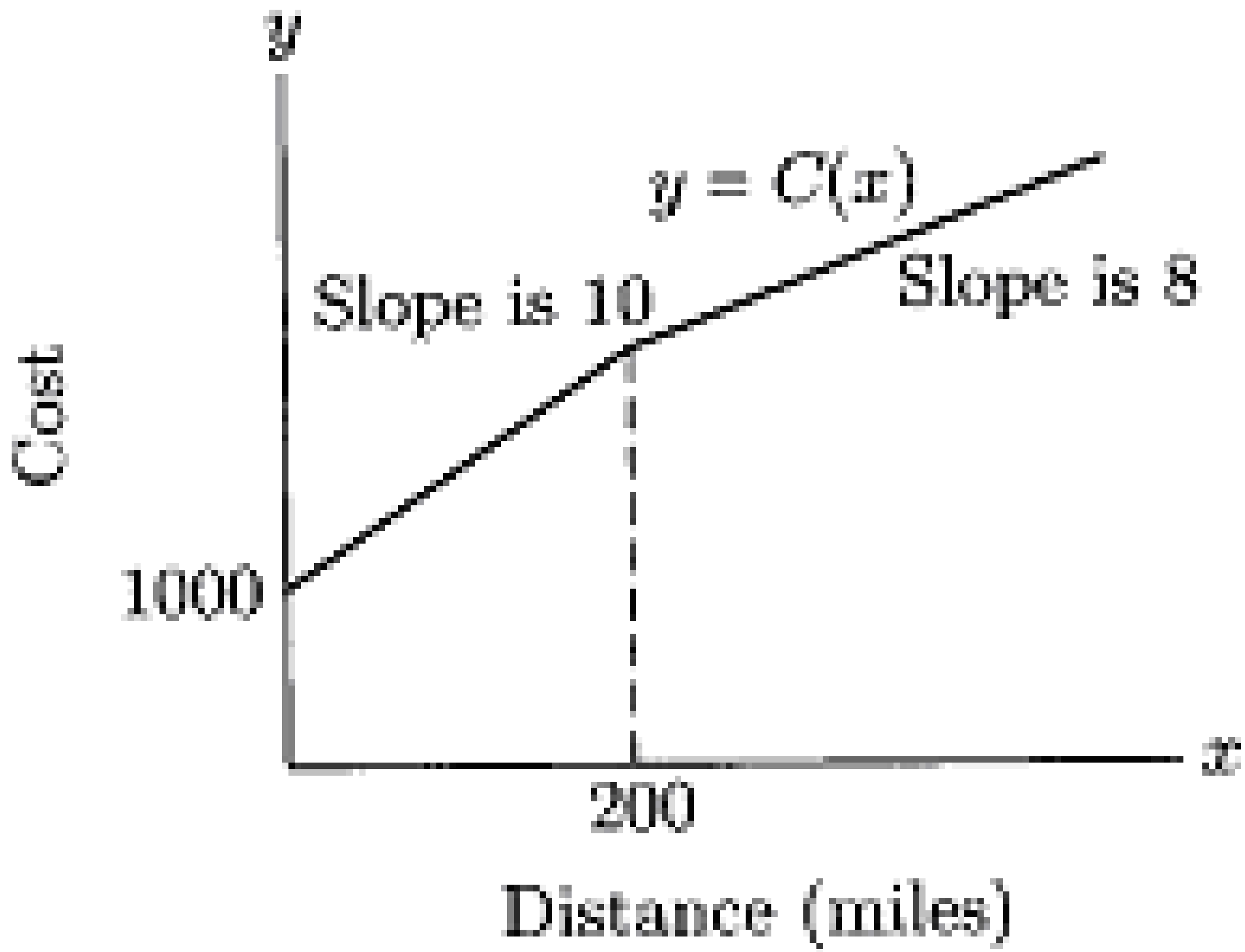
If x exceeds 200 miles, then the cost will be

$$C(x) = 3000 + 8(x - 200) = 1400 + 8x$$

So the function C(x) is given by

$$C(x) = \begin{cases} 1000 + 10x, & 0 < x \le 200 \\ 1400 + 8x, & x > 200 \end{cases}$$

The graph of C(x) is



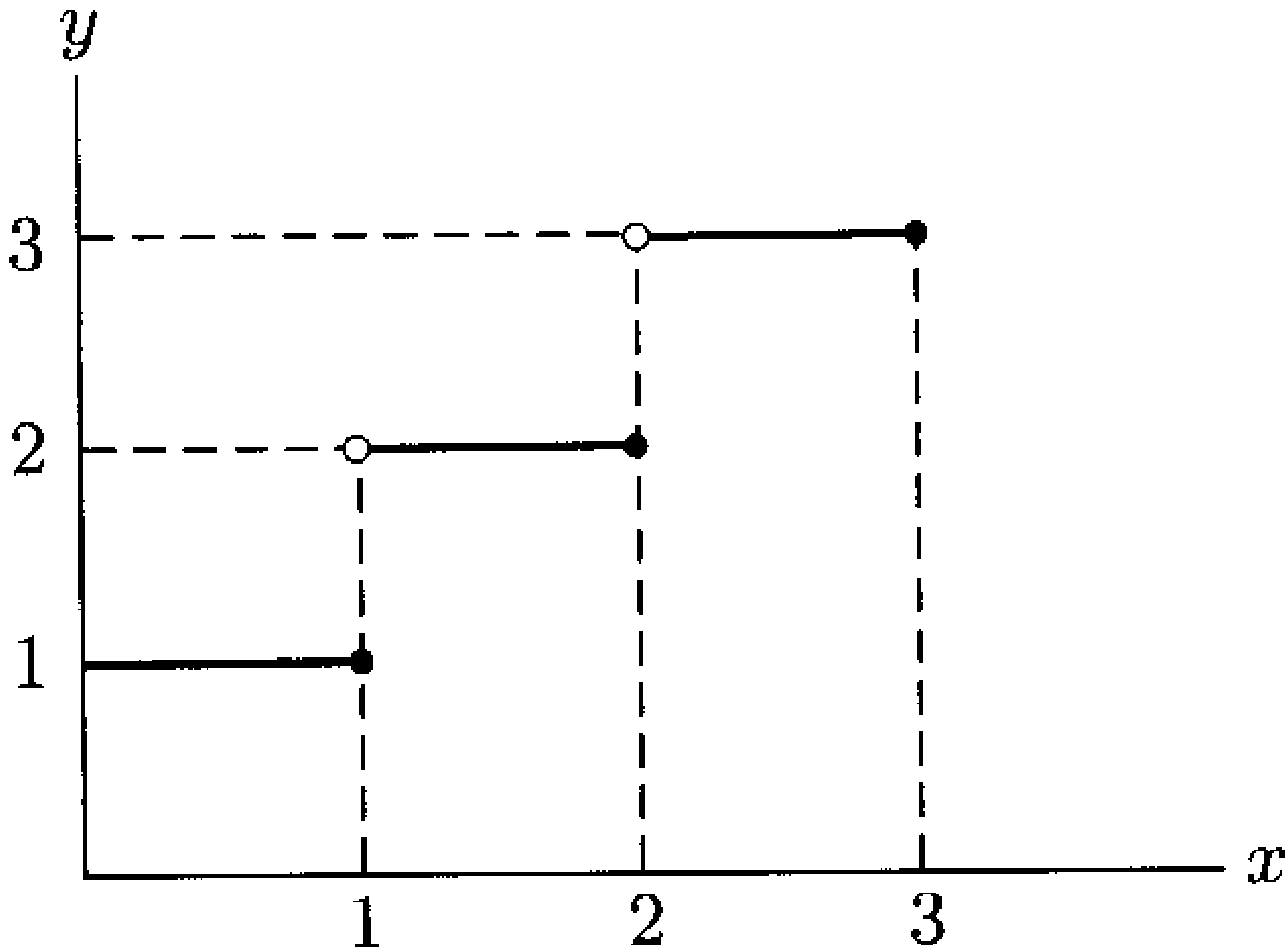
Continuity

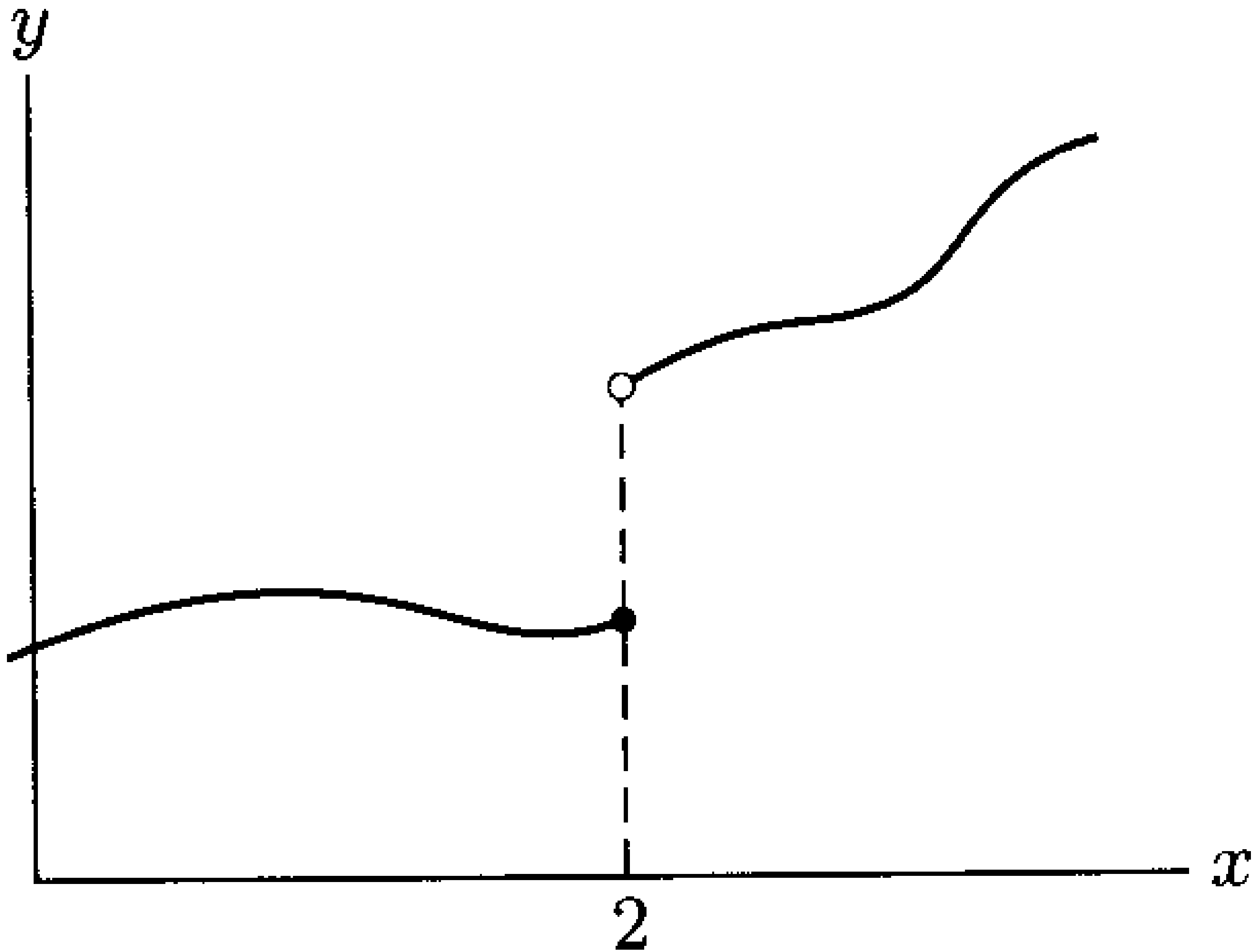
Continuity is closely related to the concept of differentiability.

We say that a function is continuous at x = a if its graph has no breaks or gaps as it passes through the point (a, f(a)).

If a function f(x) is **continuous** at x = a, it should be possible to sketch its graph without lifting the pencil from the paper at the point (a, f(a)).

The following graphs depict functions that are not continuous.


y
3
2
1
x
1
2
3


y
x
2

If f(x) is differentiable at x = a, then f(x) is continuous at x = a.

So, a function that is differentiable at x = a will be continuous at x = a.

Note however, it is still possible for a function to be continuous at x = a, but not differentiable.

Expressing continuity in terms of limits, we have the **Limit Definition of Continuity**

A function f(x) is continuous at x = a provided the following limit relation holds:

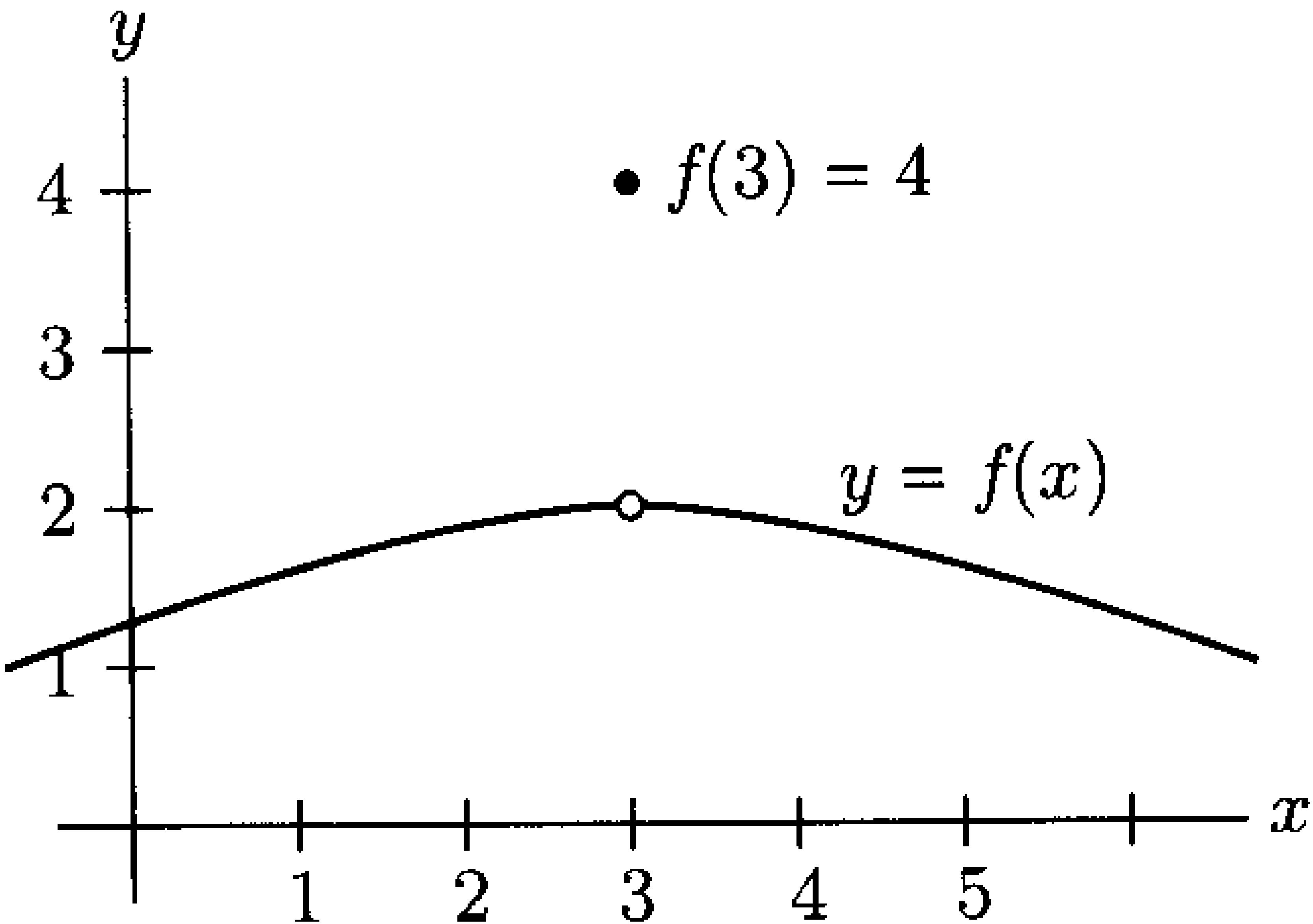
$$\lim_{x \to a} f(x) = f(a)$$

$$\lim_{x \to a} f(x) = f(a)$$

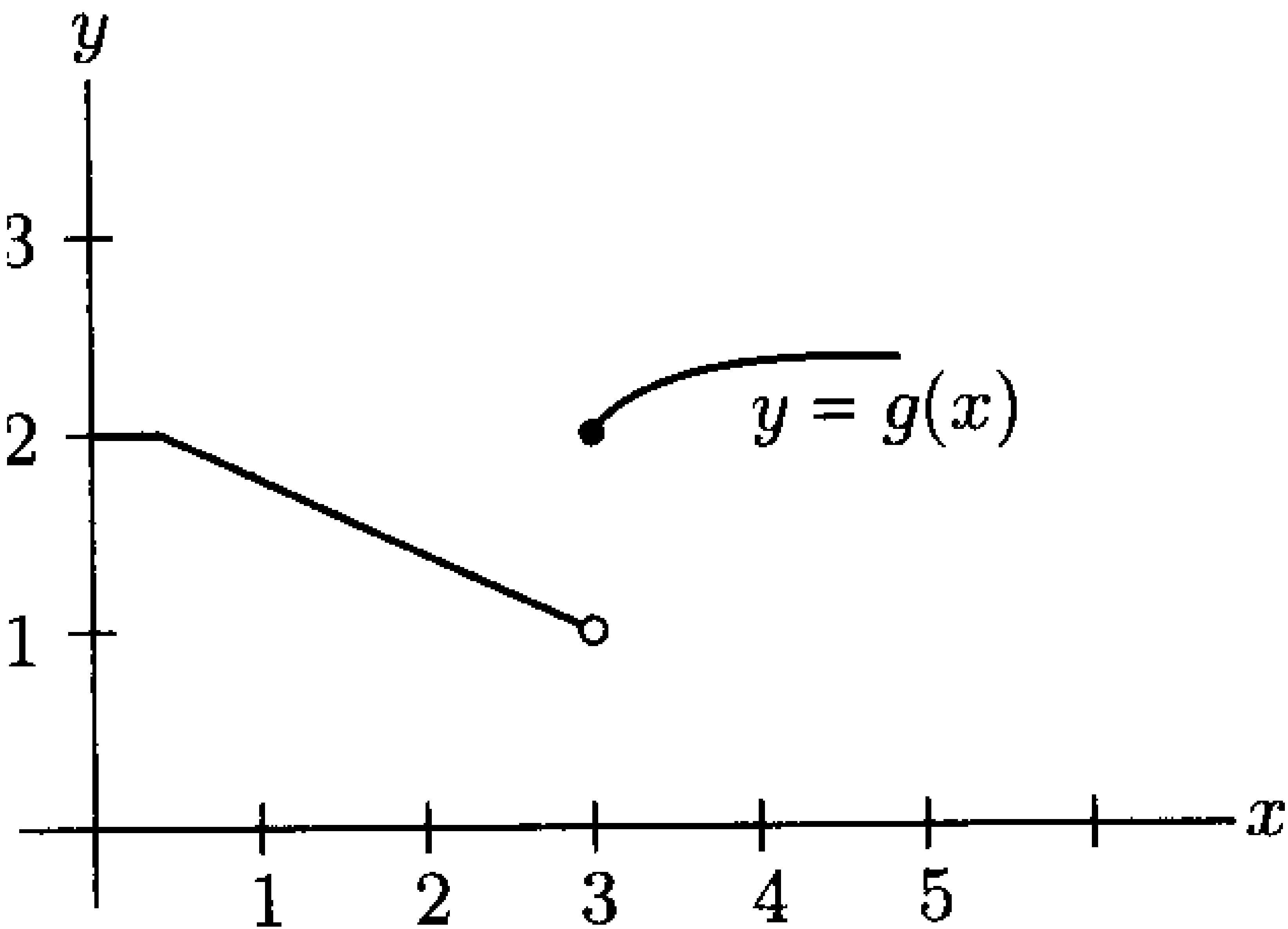
In order for this to hold, three conditions must be fulfilled.

1. ***f(x) must be defined at x = a***
2. $\lim_{x \to a} f(x)$ ***must exist***
3. ***The limit*** $\lim_{x \to a} f(x)$ ***must have the value f(a)***

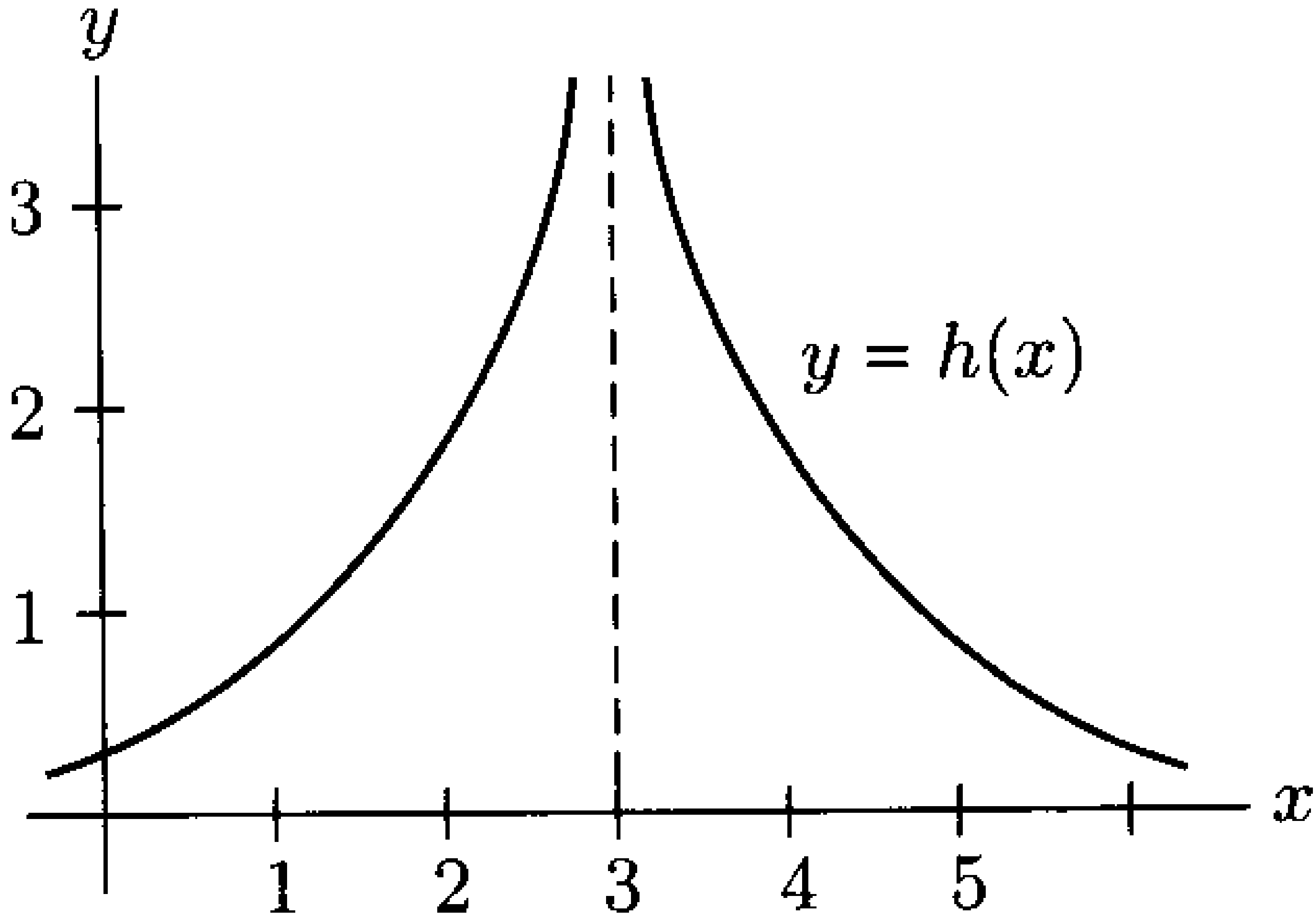
Which of the graphs represent continuous functions?

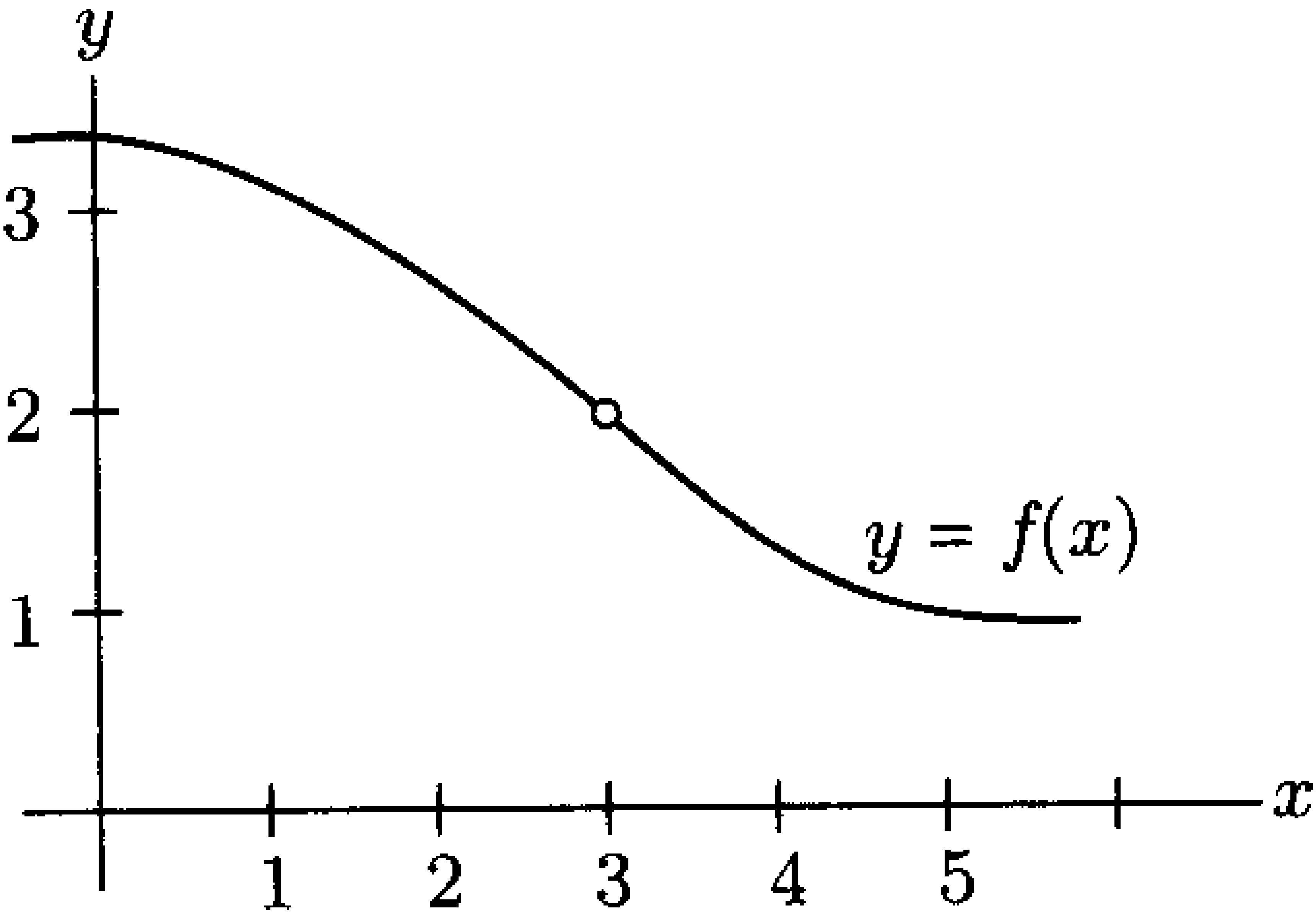


y
4
3
2
1
f(3) = 4
y = f(x)
x
1
2
3
4
5



y
3
2
1
y = g(x)
x
1
2
3
4
5


y
3
2
1
y = h(x)
x
1
2
3
4
5


y
3
2
1
y = f(x)
x
1
2
3
4
5