Mathematical

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MATRIX TRANSFORMATIONS

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1. **Translation:** A horizontal or vertical *slide* of an object. All points are displaced an equal distance.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ v \end{bmatrix}$$ for a horizontal shift of *h* and a vertical shift of *v*.

2. **Reflection:** When an object undergoes a *mirror image*. Each corresponding point of the object and the image are equidistant from the line of reflection.

Reflection about the x-axis. $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection about the y-axis. $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection about the line y = x. $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection about the line y = –x $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

3. **Rotation:** When an object is turned about a point (center) by an angle clockwise or anticlockwise. The following rotations are all about the origin.

Anticlockwise rotation of 90° $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Anticlockwise rotation of 180° $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Anticlockwise rotation of 270° $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Anticlockwise rotation of 360° (I_2) $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Anticlockwise rotation of θ° $$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

4. **Dilation:** When an object is reduced or enlarged by a scale factor.

A dilation of scale factor kh in the x axis and kv in the y axis.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} kh & 0 \\ 0 & kv \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

5. **Shear:** When an object has every point displaced the same amount in a given direction parallel to a given line. Generally parallel to the x and y axis. The fixed line is called the axis of sheer.

Sheer parallel to the x axis $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

Sheer parallel to the y axis $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

NOTE

1. Try to remember these however a quick way to find matrix transformations is to use the following trick:

We know that: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$

The image of (1 , 0) under the transformation determines the first column of the transformation matrix and the image of (0 , 1) determines the second column.

For example, if you know that the transformation is a reflection about the x axis.
(1 , 0) → (1 , 0) and (0 , 1) → (0 , -1)

hence the transformation matrix is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

2. When a shape with area A is transformed by matrix T the area of the image will be: $|\det T| \times A$

3. If Matrix T transforms point A to its image A′ then T^{-1} (The inverse of T) will transform A′ back to A. T^{-1} must exist.

4. Points are always *premultiplied* by the transformation matrix. Order of multiplication is crucial. Be careful when applying this to multiple transformations and their inverses.