Quadratics
The Big Picture

Mathematical
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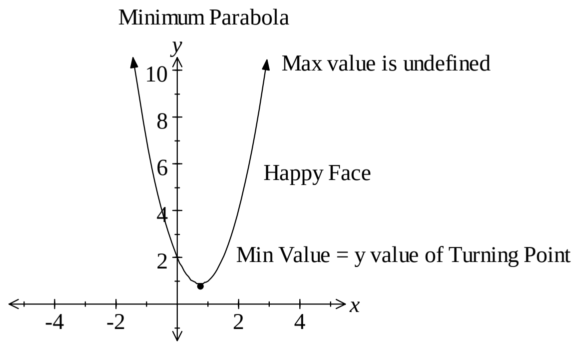
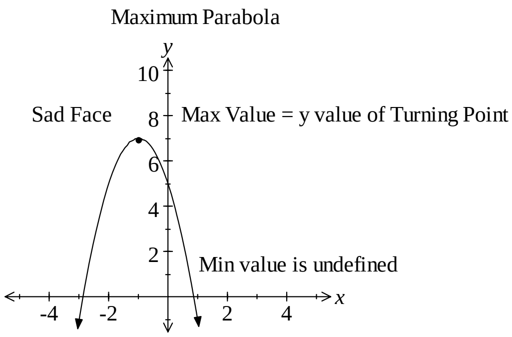
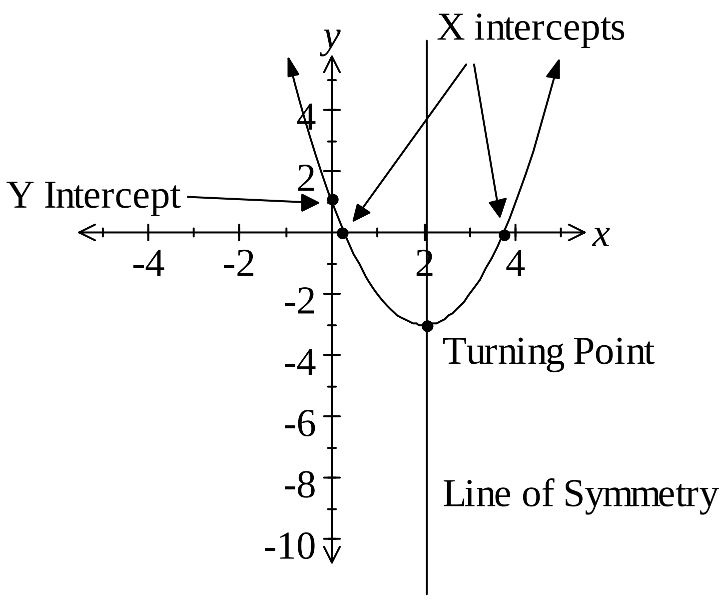
Quadratics, Trinomials, and parabola's. You have been exposed to these words frequently. This is a general summary of the many concepts you need to know and understand in relation to quadratics.

Features of a Parabola or Quadratic Graph







Standard Form

$$y = ax^2 + bx + c$$ where a, b and c are constants or numbers.

Y intercept: Occurs when $x = 0$. Always will be (0, c).
Example: $y = 2x^2 + 3x - 7$, Y intercept (0 , –7). Note the y intercept is a point.

X intercept(s): Occurs when $y = 0$. Means you have to solve the quadratic equation for x.
When solving a quadratic equation $ax^2 + bx + c = 0$, then:
Step 1: Try to factorise and then solve or
Step 2: Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: $y = x^2 + x - 6$. For x intercept(s) $x^2 + x - 6 = 0$.
Step1: $(x + 3)(x - 2) = 0$.
ie. X intercepts are (–3 , 0) and (2 , 0).
Example: $y = 2x^2 - 10x + 7$. For x intercept(s) $2x^2 - 10x + 7 = 0$
Step 1: Can't factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 2: $x = \frac{10 \pm \sqrt{100 - 56}}{4}$

$$x \approx 4.16 \text{ or } 0.84$$

ie. X intercepts are (0.84 , 0) and (4.16 , 0)

Note: You can have two, one or zero x intercept(s). See section on discriminant.

Axis or Line of Symmetry: To find the equation of the line of symmetry use the formula: $x = \frac{-b}{2a}$

Example: $y = 2x^2 - 10x + 7$ then L.O.S. $\Rightarrow$ $x = \frac{-b}{2a}$

$$x = \frac{10}{4}$$

$$x = 2.5$$

*Note: If you know the x-intercepts, and two of them exist, then the line of symmetry will be the average of the x-intercepts.

The Turning Point: To find the turning point substitute the x value of the line of symmetry back into the original equation to solve for y.

$$x = 2.5$$

Same Example: $y = 2 \times (2.5)^2 - 10 \times 2.5 + 7$

$$y = -5.5$$

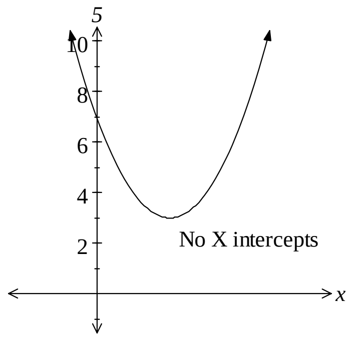
The turning point is (2.5 , –5.5)

The Discriminant: The discriminant is the part of the quadratic formula under the square root sign.

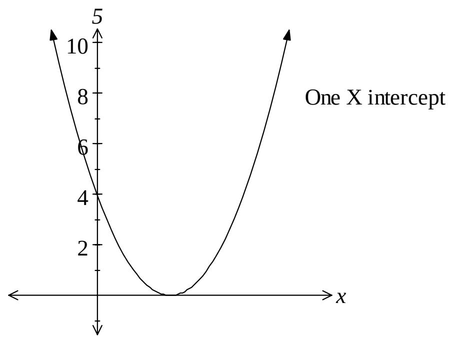
ie. The discriminant $\Delta = b^2 - 4ac$.

It is useful because it can tell us how many solutions or x intercepts belong to a quadratic equation.

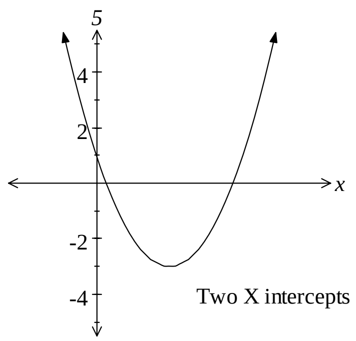
ie. For $b^2 - 4ac < 0$ we have no real solutions.
(We can't square root a negative number)



For $b^2 - 4ac = 0$ we have one real solution.
(The square root of zero is zero)



For $b^2 - 4ac > 0$ we have two real solutions.



“Completing the Square” Form

$$y = a(x - p)^2 + q$$

y intercept: Substitiute x = 0 and solve for y.
Example: $y = 2(x - 3)^2 + 4$.
For y int, x = 0. ie. $y = 2(0 - 3)^2 + 4$
$= 22$ y intercept is (0 , 22)

x intercept(s): To solve a quadratic equation in this form is quite involved so generally you wouldn’t be required to find the x intercept(s). If you are asked then one way to solve for x is to expand out the brackets and collect like terms so that you convert the equation from Complete the Square form to Standard form and then follow the steps mentioned previously.

L.O.S. and Turning Point: The big advantage of Complete the Square form is that you can instantly read from the equation the Line of Symmetry and the Turning Point.
The L.O.S. is x = p and the turning point is (p , q).
Example: $y = 2(x - 3)^2 + 4$. The L.O.S. is x = 3 and the T.P. is (3 , 4)

Transformational Approach to Graphing Quadratics:

Transforming the graph $y = x^2$

p positive graph moves ← by “p”
p negative graph moves → by “p”

$$y = a(x + p)^2 + q$$

q positive graph moves ↑ by “q”
q negative graph moves ↓ by “q”

a positive graph is “happy face”
a negative graph is “sad face”
For a between –1 and 1 “Fat graph”
$|a| < 1$
For a less than –1 and greater than 1 “Skinny graph”
$|a| > 1$

Determining Quadratics From a Table of Values

Quadratics have a "Common Second Difference" when you subtract consecutive y values. You can use important values in the table to easily determine the algebraic rule that represents the quadratic relationship. See Below:

Example:

x	0	1	2	3	4
Y	**0**	3	8	15	24

1st Difference: **3**, 5, 7, 9

Common 2nd Difference: **2**, 2, 2

c → 0 (first Y value); **a + b** → 3 (first 1st difference); **2a** → 2 (first 2nd difference)

$\therefore a = 1$, $b = 2$ and $c = 0$.
Hence the rule is $y = x^2 + 2x$ (Standard Form).

Word Problems

There are many practical word problems that may require you to determine a quadratic equation and/or interpret that equation or graph in relation to the questions. Many problems will require a sound graphical understanding of quadratics, and of the processes mentioned in this summary. You will frequently have to solve quadratic equations which is exactly the same as finding x intercepts.

Enjoy !!!!