

General capabilities

The skills, behaviours and attributes that students need to succeed in life and work in the twenty-first century have been identified in the Australian Curriculum as General capabilities. There are seven General capabilities:

- literacy
- numeracy
- competence in information and communication technology (ICT)
- critical and creative thinking
- ethical behaviour
- personal and social competence
- intercultural understanding.

The Teacher's Edition of the textbook highlights each of the General capabilities where they are relevant to the student text.

Content strands

The Australian Mathematics Curriculum is organised around three Content strands: Number and Algebra, Measurement and Geometry, and Statistics and Probability. *These strands describe what is to be taught and learnt.*

This chapter relates to the Number and Algebra strand only. It addresses the sub-strand Patterns and algebra and partially addresses the sub-strand Linear and non-linear relationships. Chapter 12 addresses other aspects of the latter sub-strand. Number and algebra concepts are *developed together, as each enriches the study of the other.*

Content descriptions

The content descriptions of the curriculum are intended to ensure that learning is appropriately ordered and that unnecessary repetition is avoided. The relevant content descriptions and their elaborations are stated under the heading *Content strands* on the following pages.

Number and algebra

6 Expressions and equations

9780170194457



Proficiencies

The Australian Curriculum: Mathematics is organised around the interaction of the three Content strands with the four proficiency strands: Understanding, Fluency, Problem solving and Reasoning.

The Teacher's Edition of this chapter highlights each of the Proficiency Strands where they are relevant to the student text. An important focus of this chapter is for students to apply their number and algebra skills through conducting investigations, solving problems, and communicating their reasoning.

Lesson ideas

The Australian Curriculum for each learning area includes annotated student work samples that illustrate the achievement standard at each year level. This chapter provides some Lesson ideas and teaching starters that teachers will find useful.



The following NelsonNet student resources are relevant to this chapter:

- Prior Learning check
- Parent Guide
- 4 Worksheets
- 6 Puzzle Sheets
- 3 Homework Units
- 4 sets of Worked Solution
- 4 sets of Extra Questions
- 5 Annotated Weblinks
- 1 Technology Activity
- 3 Animated Examples
- 1 Video Tutorial
- 1 Quiz

Also available on the NelsonNet teacher website are:

- a Curriculum guide
- several Curriculum grids
- a Teaching Plan
- a Topic Test with answers
- an ExamView question bank.

The relevant resources are indicated by the page icons.

Contents

- 6.1 Patterns and symbols
- 6.2 Number rules
- 6.3 Algebraic expressions
- 6.4 Solving equations
- Chapter summary
- Chapter review

- Prior learning Chapter 6
- Parent guide Chapter 6
- Curriculum guide Chapter 6

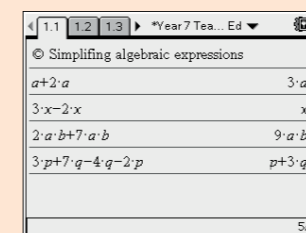
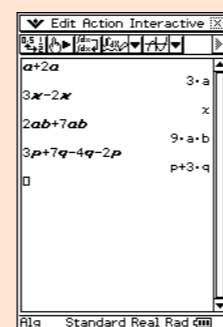
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Australian Curriculum statements

- **Patterns and algebra:** Introduce the concept of variables as a way of representing numbers using letters. Create algebraic expressions and evaluate them by substituting a given value for each variable. Extend and apply the laws and properties of arithmetic to algebraic terms and expressions.
- **Linear and non-linear relationships:** Solve simple linear equations.

Technology

CAS calculators can be used to find algebraic patterns that will help students understand algebraic expressions and the way they work. The left screen shot is from a Casio ClassPad and the right screen shot from a TI-Nspire calculator.



Cross curriculum priorities

Aboriginal and Torres Strait Islander histories and cultures

The idea of using symbols as a way of generalising relationships can be enhanced by drawing on the perspectives of Indigenous Australians. Ask students to explore Indigenous Australians' concepts of generalising by reading stories and researching on the internet.

For many Aboriginal and Torres Strait Islander children, mathematics is an intuitive process and mathematical concepts are well grounded prior to the first year of formal schooling. The curriculum recognises that mathematical ideas and procedures are used in everyday life by the majority of children and that cultural concepts can be explored and built upon through teaching and learning.

General capabilities

ICT competence

Students can *solve problems and perform onerous tasks* using a range of calculators, spreadsheets, and dynamic geometry software. It is expected that mathematics classrooms will make use of all available ICT in teaching and learning situations. However it is still important that some tasks are performed using mental skills rather than technology.

Content strands

Year 7 Number and Algebra: Patterns and algebra

Introduce the concept of variables as a way of representing numbers using letters

- understanding that arithmetic laws are powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra

Proficiencies

Understanding

Students need to *build a robust understanding of mathematical concepts that is adaptable and transferable*.

In chapter 6 students encounter the algebraic concepts behind the idea of using letters (variables) to make mathematical structures and arguments.

Example 3

Worksheet
Writing a rule
WKMNA070001

For the number pattern 3, 6, 9, 12, ...
a write a rule using a box (\square) for the term number
b work out the 8th term.

Solution

- a Write down how the pattern works. **The terms go up by 3 each time.**
Write the rule in words. **Each term is three times the term number.**
Replace the words with symbols. **The terms are $3 \times \square$.**
b Use the rule. **The 8th term = 3×8**
 $= 3 \times 8$
 $= 24$

In algebra, instead of using a box, we usually use a letter to represent a number. Then we substitute (put in) the number we want in place of the letter.

Important!

Variables and number patterns
A **variable** (pronumerals or unknown) is a letter or symbol that stands for a number. We often use the symbol n for the term number of a number pattern. The symbol a is often used for the term itself, so a_4 is the fourth term of a number pattern.

Example 4

For the number pattern 5, 10, 15, 20, ..., use a variable to:
a write a rule for the pattern
b work out the 9th term.

Solution

- a Write down how the pattern works. **The terms go up by 5 each time.**
Write the rule in words. **Each term is 5 times the term number.**
Use the symbol n for the term number. **Each term is $5 \times n$.**
b Work out the 9th term. **The 9th term, $a_9 = 5 \times 9$**
 $= 45$

Important!

Linear patterns
Linear number patterns go up or down by the same amount each time. You can write the rule using this amount multiplied by the term number. It is added or taken from the same fixed number each time.

Example 5

For the number pattern 5, 8, 11, 14, ..., use a variable to:
a write a rule for the pattern
b work out the 12th term
c find which term is equal to 32.

Solution

- a Check if the pattern is linear. **The terms go up by 3 each time, so it is linear.**
Write the multiplying part of the rule. **The rule involves $3 \times n$.**
Write the rule with ? for the fixed number. **The rule is $? + 3 \times n$.**
Try the rule for the first term. **$? + 3 \times 1 = 5$**
Work out the missing number. **? must be 2.**
Write the rule. **The terms are $a_n = 2 + 3 \times n$.**
b Work out the 12th term. **$a_{12} = 2 + 3 \times 12$**
 $= 3 + 36$
 $= 39$
c Try some numbers out to find the answer. **$2 + 3 \times 9 = 2 + 27 = 29$**
 $2 + 3 \times 10 = 2 + 30 = 32$
32 is the 10th term.
Write the answer.

Animated example
Using a variable
AEMNA070001

Puzzle sheet
Finding the term
PSMNA070001

Example 6

Write a rule in symbols for the following table of inputs and outputs.

Input p	2	3	4	5
Output v	19	16	13	10

Solution

- Write down how the pattern works. **When the input number goes up by 1, the output number goes down by 3.**
Write the rule using ? for the fixed number. **$v = ? - 3 \times p$**
Use the first term to work out ?. **$19 = ? - 3 \times 2$**
 $19 = ? - 6$
Work out ?. **? must be 25.**
Write down the rule. **$v = 25 - 3 \times p$**

Teaching notes and ideas

Barry Kissane, a lecturer at Murdoch University, has set up the website:

<http://wwwstaff.murdoch.edu.au/~kissane>

which identifies mathematical sites that are most useful for classroom teaching.

Under the heading *Learning Areas, Mathematics, Interactive Learning Websites*, or *Hot List of Maths Websites*, Kissane has collected useful and interesting classroom activities. One such activity, entitled *Baby names thru history*, links numeracy with algebra and is found at:

<http://babynamewizard.com/namevoyager>.

Students will enjoy tracking the years when their names are the most popular.

Implications for teaching, assessment and reporting

Teachers need to base their teaching on what the students already know.

Example 6 involves finding a rule using two pronumerals, p and v . The solution is based on the work done in Examples 1 through 5.

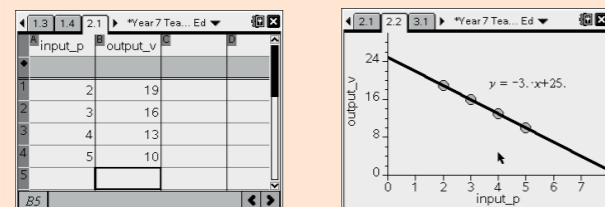
On page 4, the text states that: *A variable is a letter or symbol that stands for a number*. This definition is not given at the beginning of the section but the concept is developed implicitly in the examples leading up to Examples 6 and 7.

Technology

CAS calculators can be used to identify the rule in Example 6.

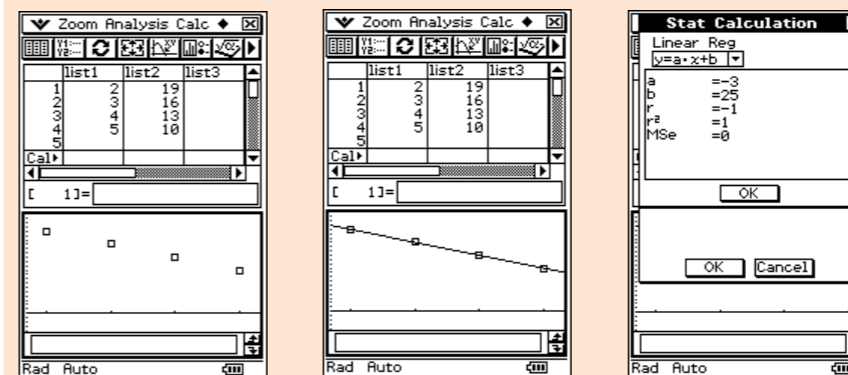
Input p	2	3	4	5
Output v	19	16	13	10

Using a TI-Nspire



The rule found using a linear regression is $v = -3p + 25$.

Using a Casio ClassPad



The rule found using a linear regression is $v = -3p + 25$.



The icons on the student pages refer to the following digital resources on the NelsonNet student website:

- **Worksheet: Writing a rule** involves identifying the rules for tabulated data.
- **Animated example: Using a variable** shows the solution steps of Example 5 through a series of animated actions.
- **Puzzle sheet: Finding the term** involves identifying terms in different series in order to decode a puzzle.

General capabilities

Numeracy

Algebra is like a written language that interprets the world around us. The ability to identify patterns is an important part of understanding that world.

Content strands

Year 7 Number and Algebra: Patterns and algebra

Introduce the concept of variables as a way of representing numbers using letters

- understanding that arithmetic laws are powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra

Proficiencies

The following proficiencies are the focus of the Exercise 6.1 problems shown.

Understanding

Students build a robust knowledge of mathematical concepts that are adaptable and transferable.

Fluency

Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately. They recall factual knowledge and concepts readily.

Technology

Using an on-line scientific calculator is an interesting way of testing number patterns.

The **Online calculators** section of the following website has a picture of a scientific calculator with 'live' buttons.

<http://calculators.torensma.net>

Students can operate the buttons using a computer pointer. A problem can be entered and the result appears in the 'live' output. Using a scientific calculator, patterns, such as the one shown opposite can be observed, then tested with new numbers and predictions made.

Example 7

Weblink
Exploring linear equations (L6553)

A large barbecue costs \$30 for 1 day's hire and \$10 for every extra day after that.
a Write a rule for the cost of hire.
b Find the cost to hire for 3 days.

Solution

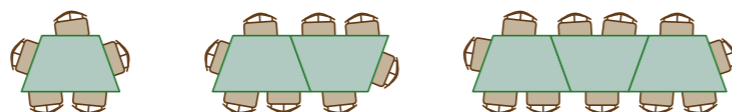
a Change the rule so the first day includes \$10. **The first day costs \$20 plus \$10.**
Now write down the rule. **It costs \$20 to hire plus \$10 for each day of hire.**
Write the rule for d days. **Hire cost = \$20 + \$10 \times d**
b Work out the cost for 3 days. **Cost for 3 days = \$20 + \$10 \times 3 = \$50**

Enter as: 20 + 10 \times 3 EXE \rightarrow 20+10x3 50

Exercise 6.1 Patterns and symbols

Understanding

1 A caterer uses portable trapezoidal tables for seating guests at functions. Tables are joined end-to-end to seat the guests as shown here.



a Extend the pattern to complete this table.

Number of tables (t)	1	2	3	4	5	6
Number of guests (g)	5	8				

- b Use the information to make a rule in words for the number of guests that can be seated.
c Write the rule using \square for the number of tables
d How many people could be seated at 6 tables?
e How many tables would need to be joined in this arrangement to seat 38 people?

See Example 1

Worked solutions

Exercise 6.1

WSMNA070001

See Example 2

2 Write a rule in words for each of the following number patterns and work out the indicated term:

- a 6, 7, 8, 9, 10, ...; 15th term b 4, 8, 12, 16, 20, ...; 21st term
c 7, 9, 11, 13, 15, ...; 9th term d 20, 19, 18, 17, ...; 11th term
e 12, 15, 18, 21, 24, ...; 12th term f 11, 8, 5, 2, -1, ...; 10th term

3 Write a rule in words for each of the following number patterns.

Input	1	2	3	4	5
Output	5	10	15	20	25

6 + 0 \times 7 =
6 + 1 \times 7 =
6 + 2 \times 7 =
6 + 3 \times 7 =
6 + 4 \times 7 =

Exercise 6.1 Answers

1 a

Number of tables (t)	1	2	3	4	5	6
Number of guests (g)	5	8	11	14	17	20

- b 2 more than 3 times the number of tables $g = 3 \times t + 2$ d 20 people e 12 tables
2 a the term number plus 5; 20 b 4 times the term number; 84 c 5 more than twice the term number; 23 d 21 minus the term number 10 e 9 more than 3 times the term

The icons on the student pages refer to the following digital resources on the NelsonNet student website:

- Weblink: Exploring linear equations** (L6553) provides an annotated link to a Learning Federation (TLE) resource which involves a general study of linear equations.
- Weblink: Exploring algebra** (L6552) provides an annotated link to a Learning Federation (TLE) resource that provides an introduction to basic algebra concepts.
- Extra questions: Exercise 6.1** includes additional questions relating to the content of section 6.1 on patterns and symbols.
- Worked solutions: Exercise 6.1** includes fully worked solutions for questions 2e, 7a and 11.

b

Input	6	7	8	9	10
Output	4	5	6	7	8

c

Input	2	3	4	9	10
Output	8	11	14	29	32

d

Input	3	6	9	12	15
Output	7	13	19	25	31

e

Input	4	7	8	9	12
Output	5	11	13	15	21

f

Input	3	8	10	9	7
Output	13	33	41	37	29

4 For the number pattern 3, 6, 9, 12, ...:

- a write a rule using a box (\square) for the term number b work out the 8th term.

5 For the number pattern 2, 7, 12, 17, ...:

- a write a rule using a box (\square) for the term number b work out the 6th term.

6 For the number pattern 7, 11, 15, 19, ...:

- a write a rule using a box (\square) for the term number b work out the 10th term.

See Example 3

7 For each of the following number patterns, use the variable n to write the rule and find the indicated term.

- a 17, 20, 23, 26, 29, ...; 10th term b 1, 3, 5, 7, 9, ...; 8th term
c 1, 5, 9, 13, 17, ...; 12th term d 18, 16, 14, 12, 10, ...; 20th term
e -3, 1, 5, 9, 13, ...; 14th term f 4, 1, -2, -5, -8, ...; 9th term

Fluency

See Examples 4, 5

Worked solutions

Exercise 6.1

WSMNA070001

See Example 6

8 Write rules in symbols for the following tables of inputs and outputs.

a

f	1	2	3	4	5	6
b	1	4	7	10	13	16

b

m	1	2	3	4	5	6
p	2	7	12	17	22	27

c

m	0	1	2	3	4	5
b	3	6	9	12	15	18

d

b	3	4	5	6	7	8
k	8	10	12	14	16	18

e

r	0	1	2	3	4	5
s	1	4	7	10	13	16

f

a	2	3	4	5	6	7
b	2	4	6	8	10	12

- number; 45 f 14 minus three times the term number; -16
3 a Five times the term number b 2 less than the term number c 2 more than 3 times the term number d 1 more than double the term number e 3 less than double the term number f 1 more than four times the term number
4 a $3 \times \square$ b 24
5 a $5 \times \square - 3$ b 27
6 a $4 \times \square + 3$ b 43
7 a $3n + 14$; 44 b $2n - 1$; 15 c $4n - 3$; 45 d $20 - 2n$; -20 e $4n - 7$; 49
f $7 - 3n$; -20
8 a $b = 3 \times f - 2$ b $p = 5 \times m - 3$ c $b = 3 \times m + 3$ d $k = 2 \times b + 2$ e $s = 3 \times r + 1$
f $b = 2 \times a - 2$

Proficiencies

The following proficiency is covered in these last few questions of Exercise 6.1.

Problem solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.

Exercise 6.1 Answers

- 9 53
- 10 18
- 11 after 29 days
- 12 \$6.80

General capabilities

Literacy

Literacy in this context can be described as interpreting mathematical texts and symbols – understanding the language of mathematics.

Content strands

F–Y10 Number and algebra

The strand statements below give an overall picture of the understandings and skills that students should be developing. Applying a range of strategies for computation and understand the connections between operations. Recognising patterns and understanding the concepts of variable and function. Building on understanding of the number system to describe relationships and formulate generalisations.

Teaching notes and ideas

Number rules

The Maths is Fun website is worth exploring. It is located at: www.mathsisfun.com/associative-commutative-distributive.html To quote from the website: *Associative, Commutative and Distributive Laws. Wow! What a mouthful of words! But the ideas are simple.*

- Problem solving**
- 9 Doreen is saying some numbers and someone hears her say '11, 14, 17, 20, 23'. What will be the 15th number she says?
- 10 'One elephant went out to play, upon a spider's web one day
He found it such tremendous fun, he called three more elephants to come
Four elephants went out to play, upon a spider's web one day
They found it such tremendous fun, they called three more elephants to come
Seven elephants went ...
...
They all fell down, and the poem is done!
The spider's web will not hold more than 20 elephants. How many lines does the poem have?
- See Example 7
- 11 A silo has 2050 kg of cattle feed. Each day, 70 kg is used to supplement the feed of a herd of cattle. How long is it before the silo needs refilling?
- Worked solutions**
Exercise 6.1
WSMNA070001
- 12 As a child in a family gets older, they get more pocket money. Peter is 6 and gets 80 cents a week. Pat is 8 and gets \$2 a week. Sue is 11 and gets \$3.80 a week. Cherie is 16. How much does she get each week?

6.2 Number rules

You already know that you can use 'turn-arounds' for multiplying and adding. The product 8×3 has the same answer as 3×8 , and $4 + 5$ has the same answer as $5 + 4$. You can always turn around addition and multiplication.

When you multiply three numbers, for example $7 \times 4 \times 5$, it doesn't matter if you do 7×4 first or 4×5 first. You would probably calculate 4×5 first to get $7 \times (4 \times 5) = 7 \times 20 = 140$ because it is easier than doing $(7 \times 4) \times 5 = 28 \times 5 = 140$. When you add three numbers you can always choose whether to start with the front numbers or the back numbers. For $123 + 27 + 48$ you would add the front numbers first to get $150 + 48$, but for $24 + 38 + 52$ you would add the back numbers first to get $24 + 90$.

Mathematicians have special names for turn-around and front-or-back-first rules.

Important!

Commutative and associative laws

Addition and multiplication are both **commutative**. The order in which the operation is done does not matter, so $a + b = b + a$ and $a \times b = b \times a$ for any numbers.

Addition and multiplication are both **associative**.

When the operation is performed on three numbers, it does not matter which pair is done first, so $a + b + c = (a + b) + c = a + (b + c)$ and $a \times b \times c = (a \times b) \times c = a \times (b \times c)$.

These rules are called **laws** because they work for all numbers.

Although $5 - 5$ can be turned around to $5 - 5$, you cannot turn $5 - 3$ around, so subtraction is not commutative. To be commutative, you must be able to change the order for *any* numbers. Division is not commutative because $6 \div 3 \neq 3 \div 6$.

Example 8

Calculate $18 \times 25 \times 40$ in your head.

Solution

Calculate 25×40 first as it is easiest.

Now calculate 18×1000 .

Write the answer.

Think $25 \times 4 \text{ tens} = 100 \text{ tens} = 1000$

Think $18 \times 1000 = 18 \text{ thousand}$

$18 \times 25 \times 40 = 18\,000$

When you have many numbers to add together, you can add them in any order because addition is both commutative and associative. It makes sense to choose the order to make tens.

Example 9

Work out $3 + 24 + 22 + 9 + 11 + 17 + 38$.

Solution

Put the units together to make tens.

$$3 + 24 + 22 + 9 + 11 + 17 + 38$$

Write down what the units add up to.

The units make 3 tens and there are 4 units left over

Add the rest of the tens in your head.

3 and 2 and 2 and 1 and 1 and 3 makes 12 tens

Write the answer.

$$3 + 9 + 22 + 17 + 24 + 11 + 38 = 124$$

Animated example

Associative law

AEMNA070002

You can calculate 36×24 in your head by multiplying 36 by 20 and then adding 36×4 . These are both easy to do using 'double and put on a zero' and 'double double' to give $720 + 144$, which is 864. You can do 47×19 by calculating 47×20 and subtracting 47×1 to get $940 - 47$, which is 893. Any multiplication can be broken into two parts like these, but you can also do the opposite when two numbers are multiplied by the same number and then added or subtracted.

$28 \times 34 + 28 \times 16$ can be calculated more easily as 28×50 . $28 \times 34 = 952$, $28 \times 16 = 448$ and $28 \times 50 = 1400$, which is indeed $952 + 448$. This works for any case, so if you have to add or subtract the product of the same number with some others, you can always perform the addition or subtraction first. Sometimes this is easier, and sometimes it's easier to do it the other way around. Mathematicians also have a special name for this rule.

Important!

Distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c$$

$$\text{and } a \times (b - c) = a \times b - a \times c$$

Links to other learning areas

English

The poem that shows mathematical patterns can also be explored in English.

*One elephant went out to play,
upon a spider's web one day
He found it such tremendous fun;
he called three more elephants to come*

*Four elephants went out to play,
upon a spider's web one day
They found it such tremendous fun;
they called three more elephants to come*

*Seven elephants went out to play,
upon a spider's web one day.....*

The Australian Concise Oxford Dictionary describes a poem as a 'metrical composition', and the word 'metrical' suggests a mathematical measuring system or of 'having a metre'. These are fascinating concepts that relate to both English and Mathematics.



The icons on the student pages refer to the following digital resources on the NelsonNet student website:

- **Worked solutions: Exercise 6.1** includes fully worked solutions for questions 2e, 7a and 11.
- **Animated example: Associative law** shows the solution steps of Example 9 through a series of animated actions.

Technology

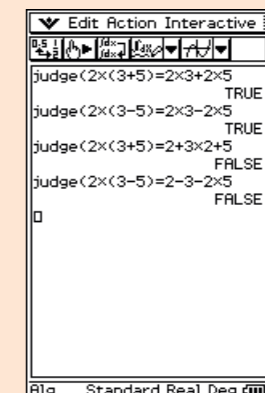
A scientific calculator can suggest patterns.

$$2 \times (3 + 5) = 2 \times 3 + 2 \times 5$$

And

$$2 \times (3 - 5) = 2 \times 3 - 2 \times 5$$

Such patterns can be tested on a CAS calculator. Students can enter right and wrong patterns and have their answer 'judged', as shown on the right.



Commutative Laws

The "Commutative Laws" say that you can swap numbers over and still get the same answer, when you add or multiply.

Sometimes it is easier to add or multiply in a different order:

What is $36 + 19 + 4$?

Consider regrouping to make the sum easier.

$$19 + 36 + 4 = 19 + 40 = 59$$

What about $2 \times 16 \times 5$?

$$2 \times 16 \times 5 = 2 \times 5 \times 16 = 10 \times 16 = 160$$